Practice #1 Solutions

1. Let N = "Talan took an afternoon nap," and let G = "Talan is grumpy now." The translation looks like:

$(N \vee G)$
G

$\sim N$

N	G	$(N \vee G)$	G	$\sim N$
1	1	1	1	0
1	0	1	0	0
0	1	1	1	1
0	0	0	0	1

The argument is invalid—owing to the first row.

2. Let P = "Candace studies," and let Q = "Candace will earn an A on her exam." The translation looks like:

$(P \rightarrow Q)$
$(\sim P \rightarrow Q)$

Q

P	Q	$(P \rightarrow Q)$	$(\sim P \rightarrow Q)$	Q
1	1	1	1	1
1	0	0	1	0
0	1	1	1	1
0	0	1	0	0

The argument is valid.

3. Conjunction Introduction

P	Q	$(P \wedge Q)$
1	1	1
1	0	0
0	1	0
0	0	0

4. Disjunction Introduction

P	Q	$(\sim P \rightarrow Q)$	$(P \vee Q)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

5. Negation Introduction

P	Q	$(P \rightarrow Q)$	$(P \rightarrow \sim Q)$	$\sim P$
1	1	1	0	0
1	0	0	1	0
0	1	1	1	1
0	0	1	1	1

6. $\{ (\sim P \rightarrow Q), (P \rightarrow (R \wedge S)), (Q \rightarrow (R \wedge S)) \} \vdash R.$

1	(1)	$(\sim P \rightarrow Q)$	A
2	(2)	$(P \rightarrow (R \wedge S))$	A
3	(3)	$(Q \rightarrow (R \wedge S))$	A
1	(4)	$(P \vee Q)$	1 ∨I
1,2,3	(5)	$(R \wedge S)$	2,3,4 ∨E
1,2,3	(6)	R	5 ∧E

7. $\{ \sim P, \sim Q, ((\sim P \wedge \sim Q) \rightarrow (\sim R \rightarrow S)) \} \vdash (S \vee R)$

1	(1)	$\sim P$	A
2	(2)	$\sim Q$	A
3	(3)	$((\sim P \wedge \sim Q) \rightarrow (\sim R \rightarrow S))$	A
1,2	(4)	$(\sim P \wedge \sim Q)$	1,2 ∧I
1,2,3	(5)	$(\sim R \rightarrow S)$	3,4 →E
1,2,3	(6)	$(S \vee R)$	5 ∨I

8. $\{ Q, ((P \rightarrow Q) \rightarrow P) \} \vdash (P \wedge Q)$

1	(1)	Q	A
2	(2)	$((P \rightarrow Q) \rightarrow P)$	A
1	(3)	$(P \rightarrow Q)$	1 →I
1,2	(4)	P	2,3 →E
1,2	(5)	$(P \wedge Q)$	1,4 ∧I

9. $\{ P, (P \rightarrow R), (Q \rightarrow R) \} \vdash R$

1	(1)	P	A
2	(2)	$(P \rightarrow R)$	A
3	(3)	$(Q \rightarrow R)$	A
1	(4)	$(\sim Q \rightarrow P)$	1 →I
1	(5)	$(P \vee Q)$	4 ∨I
1,2,3	(6)	R	2,3,5 ∨E

10. Let D = "Michael drinks," let A = "Michael is allowed to drink," let S = "Sally drinks." So, the argument is translated as:

$\{ (D \rightarrow A), (S \rightarrow D), \sim A \} \vdash \sim S$

1	(1)	$(D \rightarrow A)$	A
2	(2)	$(S \rightarrow D)$	A
3	(3)	$\sim A$	A
3	(4)	$(D \rightarrow \sim A)$	3 →I
1,3	(5)	$\sim D$	1,4 ∼I
1,3	(6)	$(S \rightarrow \sim D)$	5 →I
1,2,3	(7)	$\sim S$	2,6 ∼I