Practice #2 Solutions

1. Let K = "Katy goes to the mall," let G = "Katy's friends go to the mall," and let W = "Katy wishes she could go to the mall." The translation looks like:

$$\begin{array}{l} (\sim K \vee G) \\ (K \rightarrow \sim G) \\ (\sim K \rightarrow W) \\ \hline W \end{array}$$

K	G	W	$(\sim K \vee G)$	$(K \rightarrow \sim G)$	$(\sim K \rightarrow W)$	W
1	1	1	1	0	1	1
1	1	0	1	0	1	0
1	0	1	0	1	1	1
1	0	0	0	1	1	0
0	1	1	1	1	1	1
0	1	0	1	1	0	0
0	0	1	1	1	1	1
0	0	0	1	1	0	0

The argument is valid.

2. $\{ (\sim K \vee G), (K \rightarrow \sim G), (\sim K \rightarrow W) \} \vdash W$.

1	(1)	$(\sim K \vee G)$	A
2	(2)	$(K \rightarrow \sim G)$	A
3	(3)	$(\sim K \rightarrow W)$	A
4	(4)	G	A (for CP)
4	(5)	$(K \rightarrow G)$	4 →I
2,4	(6)	$\sim K$	2,5 ~I
2,3,4	(7)	W	3,6 →E
2,3	(8)	$(G \rightarrow W)$	4,7 CP
1,2,3	(9)	W	1,3,8 ∨E

3. Arrow Introduction

P	Q	$(P \rightarrow Q)$
1	1	1
1	0	0
0	1	1
0	0	1

4. Definition of Material Conditional

P	Q	$(\sim P \vee Q)$	$(P \rightarrow Q)$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

5-7. $\{ (\sim P \vee Q) \} \vdash (P \rightarrow Q)$.

1	(1)	$(\sim P \vee Q)$	A
2	(2)	P	A (for CP)
3	(3)	$\sim P$	A (for CP)
4	(4)	Q	A (for CP)
4	(5)	$(P \rightarrow Q)$	4 →I
	(6)	$(Q \rightarrow (P \rightarrow Q))$	4,5 CP
2	(7)	$(\sim Q \rightarrow P)$	2 →I
3	(8)	$(\sim Q \rightarrow \sim P)$	3 →I
2,3	(9)	$\sim\sim Q$	7,8 ~I
2,3	(10)	Q	9 ~E
3	(11)	$(P \rightarrow Q)$	2,10 CP
	(12)	$(\sim P \rightarrow (P \rightarrow Q))$	3,11 CP
1	(13)	$(P \rightarrow Q)$	1,6,12 ∨E

8. $\{ \sim(P \wedge Q) \} \vdash (\sim P \vee \sim Q)$.

1	(1)	$\sim(P \wedge Q)$	A
2	(2)	$\sim\sim P$	A (for CP)
3	(3)	Q	A (for reductio)
2	(4)	P	2 ~E
2,3	(5)	$(P \wedge Q)$	3,4 ∧I
2	(6)	$(Q \rightarrow (P \wedge Q))$	3,5 CP
1	(7)	$(Q \rightarrow \sim(P \wedge Q))$	1 →I
1,2	(8)	$\sim Q$	6,7 ~I
1	(9)	$(\sim\sim P \rightarrow \sim Q)$	2,8 CP
1	(10)	$(\sim P \vee \sim Q)$	9 ∨I

9. $\{ \sim(P \vee Q) \} \vdash (\sim P \wedge \sim Q)$.

1	(1)	$\sim(P \vee Q)$	A
1	(2)	$(P \rightarrow \sim(P \vee Q))$	1 →I
3	(3)	P	A (for reductio)
3	(4)	$(\sim Q \rightarrow P)$	3 →I
3	(5)	$(P \vee Q)$	4 ∨I
	(6)	$(P \rightarrow (P \vee Q))$	3,5 CP
1	(7)	$\sim P$	2,6 ∼I
1	(8)	$(Q \rightarrow \sim(P \vee Q))$	1→I
9	(9)	Q	A (for reductio)
9	(10)	$(\sim P \rightarrow Q)$	9 →I
9	(11)	$(P \vee Q)$	10 ∨I
	(12)	$(Q \rightarrow (P \vee Q))$	9,11 CP
1	(13)	$\sim Q$	8,12 ∼I
1	(14)	$(\sim P \wedge \sim Q)$	7,13 ∧I