PHIL 103: Logic and Reasoning QRII

Practice #5 Solutions

1. Sally only dates men who are shorter than she is.

Let D = "… dates ---," let M = "… is a man," let S = "… is shorter than ---," and let s = Sally. Then one possible translation is this:

$$(\forall x)(Dsx \rightarrow (Mx \wedge Sxs))$$

Possibly, the sentence is ambiguous, though. It might be read as saying that among the men that Sally dates, she only dates the short ones. But she might also date women. In that case, the correct representation should be:

$$(\forall x)((Mx \wedge Dsx) \rightarrow Sxs)$$

I had intended the first when I wrote the sentence, but the second is plausible as well.

2. If everything is bigger than something, then nothing is smaller than everything.

Let B = "… is bigger than ---," and let S = "… is smaller than ---."

$$((\forall x)(\exists y)Bxy \rightarrow \sim(\exists x)(\forall y)Sxy)$$

You might notice that the sentence above is a logical truth under the assumption that "x is not smaller than y" means "Either x is bigger than y or x is the same size as y."

3. Unless everyone studies hard, someone will be disappointed.

Let S = "… studies hard," and let D = "… will be disappointed." Then, one way to translate this is by taking "unless" to be inclusive-or in a Polish notation position. Hence,

$$((\forall x)Sx \vee (\exists x)Dx)$$

A natural alternative is to translate as:

$$(\sim(\forall x)Sx \rightarrow (\exists x)Dx)$$

By pushing through the negation across the quantifier, we get:

$$((\exists x)\sim Sx \rightarrow (\exists x)Dx)$$

Which now says, "If someone does not study hard, then someone will be disappointed." Notice that there is no requirement in the *logic* that identifies the non-studier with the one who is disappointed, although this is *implicated* in the English sentence.

4. The basic issue that I want you to puzzle about is this. It is not clear that cake, which is a *generic* noun, is a concrete individual that could be named by a constant letter. That is, "cake" is different than "Talan." If constants are names, then it is not clear that cake can be represented by a constant in our language.

5. One way to try patching up the translation is by replacing the constant c with a predicate C = "… is cake," or "… is a cake," or "… has cake-like properties," or some such and then quantifying. If one uses a universal quantifier, then one gets:

$$(\forall x)(Cx \rightarrow \sim(\exists y)Byx)$$

Problem. The translation here says that anything that is cake is at least as good as everything else. But it seems plausible that one could say, "Nothing is better than cake," while allowing exceptions. That is, plausibly, one could say, "Nothing is better than cake," while also allowing that some things are better than this or that cake – for especially badly made cakes, say. "Nothing is better than cake, but *this* cake is worse than dog biscuits!"

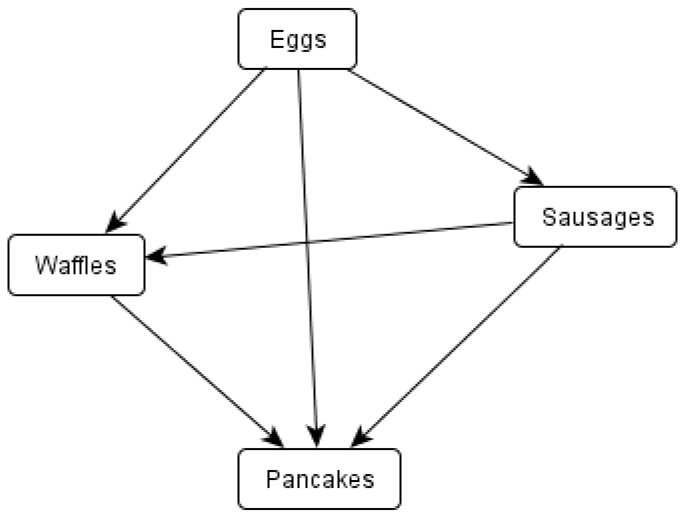
So, maybe we try an existential quantifier instead. Then one gets:

$$(\exists x)(Cx \wedge \sim(\exists y)Byx)$$

Problem. The translation here says that there is something that is cake and at least as good as anything else. But plausibly, the original English sentence was saying something stronger than that. Imagine if almost all cake was complete dreck, but one time, in Renaissance Italy, Leonardo da Vinci made a cake so good that five people lost their lives fighting over the crumbs that fell while he flew over them in his bat hang-glider enjoying his baked treasure. Plausibly, we would not say that the generic claim, "Nothing is better than cake," is true in such a world, even though there was this one cake that was phenomenal.

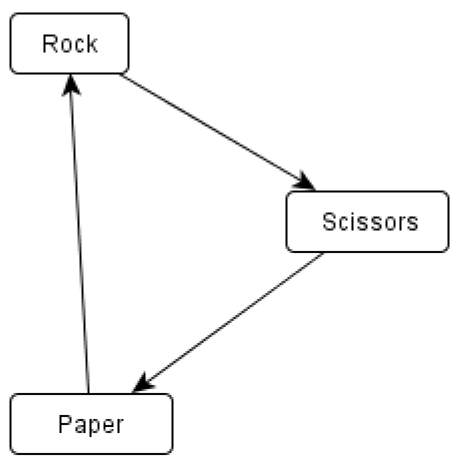
So now I'm stuck. I don't know how to go forward. Well, I have a guess about how to go forward by connecting the problem here to what is called the problem of simple generics. But *that* problem is really hard!

6. Here is a graph of Hikaru's preferences:



Hikaru's preferences are transitive, since whenever he prefers some x to some y and he prefers that y to some z, then he prefers that x to that z.

7. Here is the directed graph:



The relation is strongly irreflexive, strongly asymmetric, and strongly intransitive.

8. The simplest small world just has one object, call it a. That object has both the predicate A and the predicate B. More complicated small worlds could be given.

9. The small world described by the rock-paper-scissors picture works here. That is, if we have a world with three objects, call them a, b, and c, and the pairs <a,b>, <b,c>, and <c,a> are L-related, then it is the case that everything is L-related to something or other. But it is not the case that one thing is L-related to everything.

Another simple example is a small world with two objects, a and b where L is symmetric but strongly irreflexive. So, Lab and Lba and nothing else. In that world, nothing is L-related to everything, since no object is L-related to itself.