PHIL 103: Logic and Reasoning QRII

Practice #6

1. Translate the sentence below into our formal language. Represent as much of the internal structure of the sentence as possible.

> If nothing in Professor Livengood's office weighs more than 100 pounds and Professor Livengood can lift anything that weighs 100 pounds or less, then Professor Livengood can lift anything in his office.

2. Suppose we convert the sentence in Problem 1 into an argument of the form:

> Nothing in Professor Livengood's office weighs more than 100 pounds.
> Professor Livengood can lift anything that weighs 100 pounds or less.
> ___
> Professor Livengood can lift anything in his office.

Say whether you think the argument is valid or invalid. Give an *informal* justification for your claim, paying special attention to how the two relations, "… weighs more than 100 pounds" and "… weighs 100 pounds or less," are related to one another.

3. Describe a small world that is *not* a model for the sentence: $(\forall x)((Gx \wedge Hx) \rightarrow Rxb)$.

4. Describe a small world that is a model for the sentences: $(\exists x)(\exists y)(Bx \wedge Rxy)$, $(\exists x)Dx$, $(\exists x){\sim}Dx$, and $(\forall x)(Dx \rightarrow Bx)$.

5. Describe a small world showing that the inference below is *not* valid:

$$\{\ (\exists x)(Fx \wedge Gx)\ \} \vDash (\exists x)(Fx \wedge {\sim}Gx)$$

6. Make up predicates for F and G, and then use those predicates to translate the two sentences in Problem 5 into English.

7. For each formula ϕ in a through f below, indicate which (if any) variables occur *free*. Use a different color of ink or circle the free variables or something like that.

a. $x = y$
b. ${\sim}(Bx \rightarrow Mxy)$
c. $(Fy \wedge Gz)$
d. $(\forall x)(Fx \wedge Gy)$
e. $((\exists x)Dx \wedge Kxy)$
f. $(\forall x)(\exists y)(Wxy \vee Myz)$

8. For each formula ϕ in a through f below, write out the formula denoted by $\phi[x/d]$.

a. $(Hx \wedge (\forall y)(x = y))$
b. ${\sim}(Hx \vee Lxy)$
c. $(Fy \wedge Gz)$
d. $((\forall x)(Fx \wedge Gx) \rightarrow (Fx \vee Gx))$
e. Rxx
f. ${\sim}(\exists x)(Gx \rightarrow Lxx)$

9. Show the following: $\{ (\forall x)(Gx \wedge Rxb) \} \vdash (\forall y)(Gy \wedge Ryb)$.