PHIL 103: Logic and Reasoning QRII

Practice #6 Solutions

1. Translate the sentence below into our formal language. Represent as much of the internal structure of the sentence as possible.

> If nothing in Professor Livengood's office weighs more than 100 pounds and Professor Livengood can lift anything that weighs 100 pounds or less, then Professor Livengood can lift anything in his office.

Let p = Professor Livengood, H = "… weighs more than 100 pounds," L = "… weighs 100 pounds or less," C = "… can lift …," and O = "… is in Professor Livengood's office."

$$((\sim(\exists x)(Ox \land Hx) \land (\forall x)(Lx \rightarrow Cpx)) \rightarrow (\forall x)(Ox \rightarrow Cpx))$$

2. Suppose we convert the sentence in Problem 1 into an argument of the form:

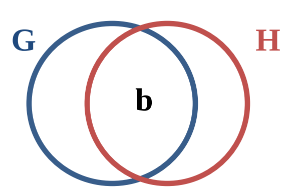
> Nothing in Professor Livengood's office weighs more than 100 pounds.
> Professor Livengood can lift anything that weighs 100 pounds or less.
>
> ---
>
> Professor Livengood can lift anything in his office.

Say whether you think the argument is valid or invalid. Give an *informal* justification for your claim, paying special attention to how the two relations, "… weighs more than 100 pounds" and "… weighs 100 pounds or less," are related to one another.

The predicate "… weighs more than 100 pounds" is complementary to the predicate "… weighs 100 pounds or less." Everything has to fall under exactly one of these two predicates. Hence, we could have expressed "Professor Livengood can lift anything that weighs 100 pounds or less" as "Professor Livengood can lift anything that does not weigh more than 100 pounds." With that re-expression, it is more obvious that the argument is valid. Nothing in Livengood's office falls into class H, which means that everything in his office falls into class L. Professor Livengood can lift anything that falls into class L. Hence, he can lift anything in his office. (The argument is unsound, however, since the first premiss is false.)

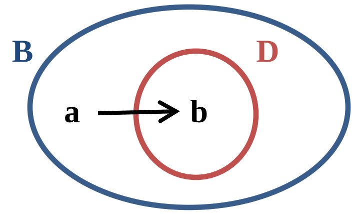
3. Describe a small world that is *not* a model for the sentence: $(\forall x)((Gx \land Hx) \rightarrow Rxb)$.

> Constants: b
> Predicates: Gb, Hb
> Relations: Nothing is R-related



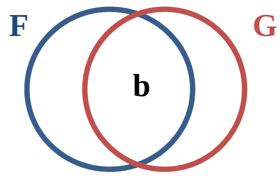
4. Describe a small world that is a model for the sentences: $(\exists x)(\exists y)(Bx \wedge Rxy)$, $(\exists x)Dx$, $(\exists x){\sim}Dx$, and $(\forall x)(Dx \rightarrow Bx)$.

Constants: a, b
Predicates: Ba, Bb, Da
Relations: Rab



5. Describe a small world showing that the inference below is *not* valid:

$$\{ (\exists x)(Fx \wedge Gx) \} \vDash (\exists x)(Fx \wedge {\sim}Gx)$$



6. Make up predicates for F and G, and then use those predicates to translate the two sentences in Problem 5 into English.

F = "… is fast."
G = "… is gigantic."

There is something that is both fast and gigantic.
There is something that is fast but not gigantic.

7. For each formula ϕ in a through f below, indicate which (if any) variables occur *free*. Use a different color of ink or circle the free variables or something like that.

Free variables are in red.

a. $x = y$
b. ${\sim}(Bx \rightarrow Mxy)$
c. $(Fy \wedge Gz)$
d. $(\forall x)(Fx \wedge Gy)$
e. $((\exists x)Dx \wedge Kxy)$
f. $(\forall x)(\exists y)(Wxy \vee Myz)$

8. For each formula ϕ in a through f below, write out the formula denoted by $\phi[x/d]$.

a. $(Hd \wedge (\forall y)(d = y))$
b. ${\sim}(Hd \vee Ldy)$
c. $(Fy \wedge Gz)$
d. $((\forall x)(Fx \wedge Gx) \rightarrow (Fd \vee Gd))$
e. Rdd
f. ${\sim}(\exists x)(Gx \rightarrow Lxx)$

9. Show the following: { (∀x)(Gx ∧ Rxb) } ⊢ (∀y)(Gy ∧ Ryb).

1	(1)	(∀x)(Gx ∧ Rxb)	A
1	(2)	(Ga ∧ Rab)	1 ∀E
1	(3)	(∀y)(Gy ∧ Ryb)	2 ∀I